International Seminar on Finite & Infinite Dimensional Dynamical Systems: Dynamics Day in Lanzhou, 2024

（2024 年兰州有限维和无限维动力系统国际研讨会）

September 2nd -6th , 2024, Lanzhou, China

Program

（程序册）

主办单位：兰州大学数学与统计学院

International Seminar on Finite & Infinite Dimensional Dynamical Systems：
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The seminar on dynamics will be held in Lanzhou (Lanzhou University) from September 2nd -6th , 2024. The aim of this workshop is to discuss the finite dimensional and infinite dimensional dynamical theories of time evolution equations, with a focus on the manifestation of finite dimensional dynamics in the theory of infinite dimensional dynamical system, while considering the essential finite dimensional dynamics of dissipative PDEs. This conference will share research experience and innovative ideas, focus on the cross integration between different directions in these fields, and determine the future development key directions.

（有限维和无限维动力系统国际研讨会将于 2024 年 9 月 2 日至 6 日在兰州（兰州大学）举行。本次研讨会的目的是研讨时间演化方程的有限维和无穷维动力学理论，重点关注有限维动力学在无限维动力系统理论中的体现，同时考虑耗散偏微分方程（无限维）的本质有限维动力学。本会议将分享研究经验和创新

思想，专注于这些领域内不同方向之间的交叉融合，确定领域内未来发展重点方向。）

Organizing Committee:

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Registration:

7:30-8:30, September 2nd, 2024, Lanzhou Hotel

(兰州饭店，2024. 09. 02, 7:30-8:30)

Conference venue:

Lecture Hall 506#, Lanzhou University Activity Center

（兰州大学大学生活动中心 506 报告厅）

Topics:

- Analysis of PDEs
- Infinite dimensional dynamical theory and applications
- Theory of dynamical chaos
- Dissipative dynamics: attractors and dimension

List of invited speakers:

Vladimir Chepyzhov

(The Institute for Information Transmission Problems, Russia)

Sergey Gonchenko

(Lobachevsky State University of Nizhny Novgorod, Russia)

Efrosiniia Karatetskaia

(University Higher School of Economics, Nizhnij Novgorod, Russia)

Alexey Kazakov

(University Higher School of Economics, Nizhnij Novgorod, Russia)

Alexey Ilyin

(Keldysh Institute of Applied Mathematics, Moscow, Russia)

Dmitrii Mints

(Imperial College London, London, UK)

Olga Pochinka

(University Higher School of Economics, Nizhnij Novgorod, Russia)

Klim Safonov

(National Research University Higher School of Economics, Russia)

Dmitry Turaev

(Imperial College London, UK)

Sergey Zelik

(Zhejiang Normal University, China & Univerity of Surrey, UK)

Academic Program

Day 1. Monday, September 2nd

9:00 — 9:30	**Registration, opening**
9:30 — 10:30	Vladimir Chepyzhov, Global and trajectory attractors for dissipative partial differential equations. **Lecture 1.** Dynamical semigroups and global attractors
10:30 — 11:30	Vladimir Chepyzhov, Global and trajectory attractors for dissipative partial differential equations. **Lecture 2.** Application to evolution equations(ODEs and PDEs).
11:30 — 14:30	**Lunch, discussions**
14:30 — 15:30	Alexey Ilyin, Inequalities for orthonormal families and optimal bounds for the dimension of attractors of dissipative dynamical systems. **Lecture 1.**
15:30 — 16:00	**Coffee break**

16:00 — 17:00 Sergey Zelik, Multi-Vortices and lower bounds for attractors' dimension in hydrodynamics (joint with A. Ilyin, A. Kostianko and D. Stone). **Lecture 1.**

17:00 — 18:00 Sergey Zelik, Multi-Vortices and lower bounds for attractors' dimension in hydrodynamics (joint with A. Ilyin, A. Kostianko and D. Stone). **Lecture 2.**

18:00 — … **Welcome party**

Day 2. Tuesday, September 3rd

9:30 — 10:30 Vladimir Chepyzhov, Global and trajectory attractors for dissipative partial differential equations. **Lecture 3.** Trajectory attractors for ODEs without uniqueness.

10:30 — 11:30 Vladimir Chepyzhov, Global and trajectory attractors for dissipative partial differential equations. **Lecture 4.** Theory of trajectory attractors and applications to PDEs (reaction-diffusion systems, 3D Navier-Stokes equations).

11:30 — 14:30 **Lunch, discussions**

14:30 — 15:30 Alexey Ilyin, Inequalities for orthonormal families and optimal bounds for the dimension of attractors of dissipative dynamical systems. **Lecture 2.**

15:30 — 16:00 **Coffee break**

16:00 — 17:00 Sergey Zelik, Multi-Vortices and lower bounds for attractors' dimension in hydrodynamics (joint with A. Ilyin, A. Kostianko and D. Stone). **Lecture 3.**

17:00 — 18:00 Sergey Zelik, Multi-Vortices and lower bounds for attractors' dimension in hydrodynamics (joint with A. Ilyin, A. Kostianko and D. Stone). **Lecture 4.**

Day 3. Wednesday, September 4th

9:30 — 10:30 Klim Safonov, Dynamical properties and topological structure of the Lorenz attractor. **Lecture 1.**

10:30 — 11:00 **Coffee break**

11:00 — 12:00 Dmitry Turaev, Fermi Acceleration and Ergodicity. **Lecture 1.**

12:00 — 13:00 Dmitry Turaev, Fermi Acceleration and Ergodicity. **Lecture 2.**

13:00 — 18:00 **Lunch, discussions**

18:00 — … **Conference dinner**.

Day 4. Thursday, September 5th

9:30 — 10:30 Olga Pochinka, Classification of three-dimensional Pixton homeomorphisms. **Lecture 1.**

10:30 — 11:30 Olga Pochinka, Classification of three-dimensional Pixton homeomorphisms. **Lecture 2.**

11:30 — 14:30 **Lunch, discussions**

14:30 — 15:30 Dmitrii Mints, Dynamics of maps with homoclic tangencies. **Lecture 1.**

15:30 — 16:00 **Coffee break**

16:00 — 17:00 Klim Safonov, Dynamical properties and topological structure of the Lorenz attractor. **Lecture 2.**

17:00 — 18:00 Klim Safonov, Dynamical properties and topological structure of the Lorenz attractor. **Lecture 3.**

Day 5. Friday, September 6th

9:30 — 10:30 Sergey Gonchenko, Dynamical chaos and bifurcation theory. **Lecture 1.**

10:30 — 11:30 Sergey Gonchenko, Dynamical chaos and bifurcation theory. **Lecture 2.**

11:30 — 14:30 **Lunch, discussions**

14:30 — 15:30 Dmitrii Mints, Dynamics of maps with homoclic tangencies. **Lecture 2.**

15:30 — 16:00 **Coffee break**

16:00 — 17:00 Alexey Kazakov and Efrosiniia Karatetskaia, On pseudohyperbolic attractors. **Lecture 1.**

17:00 — 18:00 Alexey Kazakov and Efrosiniia Karatetskaia, On pseudohyperbolic attractors. **Lecture 2.**

Global and trajectory attractors for dissipative partial differential equations

Vladimir Chepyzhov

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This mini-course is devoted to the theory of trajectory attractors of dissipative partial differential equations (PDEs).Many important problems arising in mechanics and physics lead to the study of complicated evolution PDEs and especially to the study of their solutions as time tends to infinity. For the last 5 decades, the considerable progress in solving such problems has been achieved using the theory of infinite dimensional dynamical systems and their attractors. The classical approach suggests to consider the dynamical semigroup in the phase space of initial data of the PDE under the study, which is a Hilbert or Banach space. After that, one looks for a

global attractor of this semigroup. However, to construct such single-valued semigroup the involved Cauchy problem must be uniquely solvable on arbitrary long time interval. If the solution is not unique, or the uniqueness theorem is unknown, then the classical approach is not directly applicable. Recall, there are many important PDEs for which that is the case. For example, the famous 3D Navier-Stokes system is a bounded domain. To overcome this drawback of the classical theory, one can use the theory of so-called trajectory dynamical system and their trajectory attractors developed in the works of Mark Vishik and Vladimir Chepyzhov, presented in this mini-course. In two starting lectures we will shortly expose the classical approach with application to dissipative evolution equations (ODEs and PDEs), such as reaction-diffusion systems and 2D Navier-Stokes system. In lecture 3, we explain the method of trajectory dynamical systems using quite simple but substantial system of ODEs without uniqueness of their solutions. In lecture 4, we explain how this theory is applicable to more complicated reaction-diffusion systems and

to inhomogeneous 3D Navier-Stokes system in a bounded domain, for which, as has been shown in the recent works, the long standing hypothesis of the uniqueness is closed to be refute. So, the trajectory attractors are very important in the study of long time behaviour for solutions of this and other PDEs without uniqueness.

Dynamical chaos and bifurcation theory

Sergey Gonchenko

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The theory of dynamical chaos is an interdisciplinary science that studies complex regimes demonstrated by physical, biological, chemical, etc. models of natural science and engineering using various methods (sometimes scientific and sometimes heuristic and not very rigorous). The mathematical theory of dynamical chaos is aimed at explaining these phenomena primarily in mathematical language, as well as at developing all kinds of tools for studying chaos. One of the main tasks of this theory is to explain various mechanisms of chaos appearance, in particular, during a transition from simple regimes, stationary, periodic, quasiperiodic, etc. Moreover, a major role here is played by the theory of bifurcations, which is a branch of the qualitative theory of dynamical systems that studies phenomena of rearrangements of phase portraits of dynamical systems when parameters change. In these lectures, we will try to give some overview of the fundamental results in the theory of dynamical systems and, in particular, in the bifurcation theory, which, in essence, led to the discovery of dynamical chaos and its three forms. The first two of them are classical ones: conservative chaos and dissipative chaos (strange attractors) and the third one is completely new one, which is called mixed dynamics. These lectures will also include a small historical part including a discussion on:

- How A. Poincare “accidentally” discovered conservative chaos;
- How J. Hadamard, D. Birkhoff and others came up with symbolic and topological dynamics to explain chaos;
- How Van der Pol and Van der Mark “listened” to chaos in the late 1920s;
- How A. Andronov, A. Witt and L. Pontryagin developed their theory of statistical description of dynamical systems which were 50 years ahead of the modern science;
- How the meteorologist E. Lorenz discovered the famous strange attractor, which no one knew about for a long time, and which was later named after him.

The main part of the lectures will be devoted to the issues of the modern mathematical theory of dynamical chaos and a discussion of the role played by bifurcation theory in this theory. In particular, we will show how new scenarios of the emergence of strange attractors of various types (mainly the so-called homoclinic attractors) in the case of multidimensional maps and flows were constructed using the methods of 4 the bifurcation theory. Examples of the implementation of these scenarios in specific models will be given. The discovery of mixed dynamics turned out to be completely unexpected for the author, since it is made in the process of routine study of bifurcations of homoclinic tangencies. This is, probably, the most incomprehensible and uninteresting (from the point of view of applications) part of bifurcation theory, since it deals with very interesting effects of dynamics, but on extremely small scales. However, the science is driven by curiosity and investigating the things, which are just interesting, mysteriously leads to discovery of new concepts and methods useful for applications: "The search for the beautiful leads us to the same choice as the search for the useful." (H. Poincare). In particular, the study of homoclinic tangencies led to the discovery of that type of dynamical chaos where attractor intersects with repeller and which is already detected in a number of non-trivial and important examples.. How this intersection can be possible at all will also be explained in the lectures.

Inequalities for orthonormal families and optimal bounds for the dimension of attractors of dissipative dynamical systems

Alexey Ilyin

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Estimates for the fractal dimension of the global attractors of dissipative evolution PDEs are traditionally related with the number of the degrees of freedom involved in the description of the long-time behaviour of the solutions. The dimension estimates, in turn, are based on the bounds for the N-traces of the linearized evolution overator. Therefore inequalities for systems that are orthonormal with respect to the underlying Hilbert phase space naturally come into play.

In the case of the 2D Navier–Stokes equations inequalities for the L^2-orthonormal systems of vector functions (the celebrated Lieb–Thirring inequalities) play the esential role in finding good or even optimal estimates for the dimension of the global attractors. We review a few classical and new results for certain models in mathematical fluid mechanics both in 2D and 3D.

Another popular example of an equation served by the attractor theory is a weakly damped nonlinear hyperbolic system. Here the key role is played by the inequalities for systems with orthonormal gradients. Based on them, we prove an explicit estimate for the fractal dimension of the attractor. Remarkably, the case of the spatial dimension $d \geq 3$ is simpler and in the case of a system with non-gradient perturbation the upper bound for the fractal dimension is supplemented with the lower bound of the same order in the limit of a small damping coefficient. The lower dimensional case is surprisingly more difficult, less complete, and requires a rather different technique.

On pseudohyperbolic attractors

Alexey Kazakov, Efrosiniia Karatetskaia

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One of the most fundamental problems in multidimensional chaos theory is the study of strange attractors which are robustly chaotic (i.e., they remain chaotic after small perturbations of the system). It was hypothesized in [1] that the robustness of chaoticity is equivalent to the pseudohyperbolicity of the attractor. Pseudohyperbolicity is a generalization of hyperbolicity. The main characteristic property of a pseudohyperbolic attractor is that each of its orbits has a positive maximal Lyapunov exponent. In addition, this property must be preserved under small perturbations. The foundations of the theory of pseudohyperbolic attractors were laid by Turaev and Shilnikov, who showed that the class of pseudohyperbolic attractors, besides the classical Lorenz and hyperbolic attractors, also includes wild attractors which contain orbits with a homoclinic tangency.

In this lectures, using the pseudohyperbolicity notion, we will explain how to check whether the attractor is robustly chaotic or not. We will describe the corresponding numerical methods and apply them for the study of model systems (the Lorenz, Lyubimov-Zaks, and Shimizu-Morioka systems) as well as systems arising in applications (optical laser model, model of thermal convection, ensembles of oscillators, etc.).

This work is supported by the project "Mirror Laboratories" HSE University.

Dynamics of maps with homoclinic tangencies

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The major fact in the theory of dynamical systems is that structurally unstable maps can form open regions in the space of smooth maps. The simplest mechanism of destroying structural stability is a homoclinic tangency, i.e., a non-transverse intersection of the stable and unstable manifolds of a saddle periodic orbit. By the celebrated Newhouse Theorem, in the space of smooth maps there exist open regions (Newhouse domain) where maps with homoclinic tangencies are dense. Generically, maps from the Newhouse domain exhibit extremely complicated dynamics which includes coexistence of infinitely many sinks, superexponential growth of the number of periodic points, universal dynamics. In the lectures we will give an overview of classical results on the dynamics of maps from the Newhouse domain, as well as recent advances in this area.

Classification of three-dimensional Pixton homeomorphisms

Olga Pochinka

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The course examines the so-called regular homeomorphisms on closed manifolds. The chain-recurrent set of such discrete dynamical systems consists of a finite number of hyperbolic periodic points and they are a generalization of Morse-Smale diffeomorphisms. We will study the embedding topology of their invariant manifolds, as well as their asymptotic behavior. Let's make sure that regular homeomorphisms that do not have saddle points exist only on the sphere and all such homeomorphisms are pairwise topologically conjugate. Next, a complete topological classification of three-dimensional Pixton homeomorphisms – homeomorphisms with a single saddle orbit will be presented. The space of such homeomorphisms decomposes into a countable number of classes of topological conjugacy and the Hopf knot is a complete invariant for them. We show how an arbitrary Hopf knot is realized by a Pixton homeomorphism, in particular a homeomorphism with wildly embedded invariant manifolds of saddle points. An immediate consequence of the implementation will be the proof of the fact that the supporting manifold for Pixton homeomorphisms is a 3-sphere.

Dynamical properties and topological structure of the Lorenz attractor

Klim Safonov

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The Lorenz attractor is one of the first and main examples of chaotic attractors that are robust with respect to small perturbations of a system. The robustness property is due to the presence of a hyperbolic structure in the attractor. The hyperbolicity allows to do quite extensive analysis of the topological and dynamical properties of the Lorenz attractor. In these lectures, we will discuss some definitions and geometric models of the Lorenz attractor, its topological structure and some bifurcations leading to the birth of the attractor.

This work is supported by the project "Mirror Laboratories" HSE University.

Fermi acceleration and ergodicity

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In slow-fast Hamiltonian systems the energy flow between the slow and fast degrees of freedom is impeded when the fast subsystem is ergodic, because of the existence of an adiabatic invariant, Gibbs volume entropy

of the fast subsystem. When the fast subsystem is not ergodic, the energy flow from the slow to the fast degrees of freedom is anomalously strong.

Multi-Vortices and lower bounds for attractors' dimension in hydrodynamics (joint with A. Ilyin, A. Kostianko and D. Stone)

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We present the new machinery for estimating the fractal dimension of global attractors for 2D hydrodynamics, which is not based on the Kolmogorov flows and works in the case of the whole space or in bonded domains with Dirichlet boundary conditions. The role of Kolmogorov flows is played by the sums of spatially separated unstable vortices and the machinery extends the results previously known only for periodic boundary conditions to a wide class of Navier-Stokes type problems in bounded and unbounded domains.